20857212

a) $x(t) = e^{3t} \sin(4t)$

$y(t) = e^{3t} \cos(4t)$

$z(t) = 0$

$x^2(t) + y^2(t) = e^{6t} \sin^2(4t) + e^{6t} \cos^2(4t)$

$= e^{6t}(\sin^2(4t) + \cos^2(4t))$

$= e^{6t}(1)$

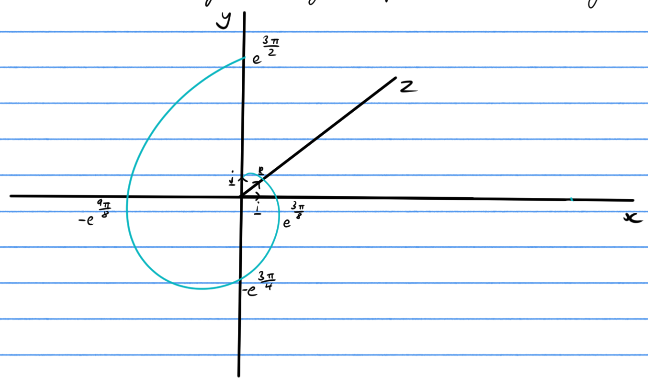
$= e^{6t}$

This is the equation of a spiral which has an exponentially increasing radius as its angle increases.

Therefore this is a logarithmic spiral.

When $t=0$, $x(t)=0$, $y(t)=1$

As t begins to increase, the $\sin(4t)$ in $x(t)$ begins to increase therefore the spiral is clockwise about the origin



b)

$\frac{d\underline{r}}{dt} = \frac{d}{dt}\left(e^{3t}\sin(4t)\right)\underline{i} + \frac{d}{dt}\left(e^{3t}\cos(4t)\right)\underline{j} + \frac{d}{dt}(0)\underline{k}$

$= \left(e^{3t}(4\cos(4t)) + 3e^{3t}\sin(4t)\right)\underline{i} + \left(e^{3t}(-4\sin(4t)) + 3e^{3t}\cos(4t)\right)\underline{j} + 0\underline{k}$

$= e^{3t}\left(4\cos(4t) + 3\sin(4t)\right)\underline{i} + e^{3t}\left(3\cos(4t) - 4\sin(4t)\right)\underline{j} + 0\underline{k}$

$\left|\frac{d\underline{r}}{dt}\right| = \sqrt{e^{6t}(4\cos(4t) + 3\sin(4t))^2 + e^{6t}(3\cos(4t) - 4\sin(4t))^2}$

$= \sqrt{e^{6t}\left[(4\cos(4t) + 3\sin(4t))^2 + (3\cos(4t) - 4\sin(4t))^2\right]}$

$= \left(e^{6t}\left[16\cos^2(4t) + 24\sin(4t)\cos(4t) + 9\sin^2(4t) + 9\cos^2(4t) - 24\sin(4t)\cos(4t) + 16\sin^2(4t)\right]\right)^{\frac{1}{2}}$

$= \left(e^{6t}\left[25\cos^2(4t) + 25\sin^2(4t)\right]\right)^{\frac{1}{2}}$

$= (25e^{6t})^{\frac{1}{2}}$

$= 5e^{3t}$

Then for the arc length across $t \in [0, 2\pi]$, s,

$s = \int_0^{\frac{\pi}{2}} \left|\frac{d\underline{r}}{dt}\right| . dt$

$= \int_0^{\frac{\pi}{2}} 5e^{3t} . dt$

$= \left[\frac{5}{3}e^{3t}\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{5}{3}e^{\frac{3\pi}{2}}\right] - \left[\frac{5}{3}\right]$

$= \frac{5}{3}\left(e^{\frac{3\pi}{2}} - 1\right)$

c) $\underline{r}(t) \cdot \frac{d\underline{r}}{dt} = e^{3t}\sin(4t)\left(e^{3t}(4\cos(4t) + 3\sin(4t))\right) + e^{3t}\cos(4t)\left(e^{3t}(3\cos(4t) - 4\sin(4t))\right) + 0$

$= e^{6t}\left(4\cos(4t)\sin(4t) + 3\sin^2(4t) + 3\cos^2(4t) - 4\cos(4t)\sin(4t)\right)$

$= e^{6t}\left(3\sin^2(4t) + 3\cos^2(4t)\right)$

$= 3e^{6t}$

$|\underline{r}(t)| = \left(e^{6t}\sin^2(4t) + e^{6t}\cos^2(4t)\right)^{\frac{1}{2}}$

$= \left(e^{6t}(\sin^2(4t) + \cos^2(4t))\right)^{\frac{1}{2}}$

$= (e^{6t})^{\frac{1}{2}}$

$= e^{3t}$

By definition of the dot product,

$\underline{r}(t) \cdot \frac{d\underline{r}}{dt} = |\underline{r}(t)|\left|\frac{d\underline{r}}{dt}\right|\cos\theta$ where θ is the angle between $\underline{r}(t)$ and $\frac{d\underline{r}}{dt}$.

$\Rightarrow 3e^{6t} = e^{3t}(5e^{3t})\cos\theta$

$\Rightarrow 3e^{6t} = 5e^{6t}\cos\theta$

$\Rightarrow \cos\theta = \frac{3}{5}$ as $e^{6t} > 0$

Hence θ is a constant.